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The Henderson Approximation and the Mass Action Law of Guldberg and Waage

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Abstract: Using contemporary accounts, we describe the scope of the Henderson approximation, and its relationship to the mass action law of Guldberg & Waage. The Henderson approximation, apparently written in logarithmic form by Hasselbalch, has the same *form* as the mass action law, but has a completely different meaning and a much more restricted range of applicability. The Guldberg-Waage law is the fundamental mass action relationship valid for all chemical equilibria, whereas the Henderson approximation is useful only within a limited range of a sufficiently concentrated two-component buffer mixture.

Between 1864 and 1879 Guldberg and Waage formulated the mass action law, one of the basic laws of chemical equilibrium. This law was crucial to the subsequent development of both physical and analytical chemistry. This year, a century after the death of Guldberg, that law remains one of the foundations of modern chemistry, and part of all chemistry curricula.

However, during the past few decades, the names of Guldberg and Waage have been replaced in many general and analytical textbooks by those of Henderson and Hasselbalch. One might argue that this is merely a matter of name giving, but unfortunately that is not the case. Henderson did make a useful contribution related to the application of the mass action law to simple acid–base buffers, a contribution that used the same *formalism* but had an entirely different *meaning*, and a much more restricted *applicability*. Henderson's approximation was much more limited in scope, and subsequent developments of the concept of buffer action, and especially the increasing use of computers, have decreased its practical importance.

Unfortunately, associating Henderson's approximation with the general law given by Guldberg and Waage has led to much unnecessary confusion, so that, in the recent words of Po and Senozan [1], "certain subtleties of the Henderson–Hasselbalch equation have become lost and the distintion between exact and approximate results has blurred."

In this note we will try to set the record straight by letting the historical record speak for itself. Specifically, we will show that (1) Henderson was well aware of the mass action law, and (2) did not claim to have originated it, but that, instead, (3) his contribution dealt with an *approximation* to that law, useful for some simple acid–base buffers over a limited pH range. The above-mentioned paper by Po and Senozan is a clear example of the confusion that can result when these matters are, in Henderson's words, "somewhat befogged."

The claim that Henderson originated the mass action law is usually based on two papers [2, 3], and we will, therefore, consider these papers first. In the second paragraph of his paper "Concerning the relationship between the strength of acids and their capacity to preserve neutrality," Henderson [2] states that

> According to the mass law, in the pure solution of a weak acid, *HA*, the relationship holds,—
>
> $$k \cdot (\mathrm{HA}) = (\overset{+}{\mathrm{H}}) \cdot (\bar{\mathrm{A}})$$
>
> where *k* is the ionization constant of the acid and the enclosed quantities represent the concentrations of the respective substances. This equation may be more conveniently written
>
> $$(\overset{+}{\mathrm{H}}) = k \cdot \frac{(\mathrm{HA})}{(\bar{\mathrm{A}})}$$

From this quote it is clear that Henderson knew the mass (action) law. In fact, the above, matter-of-fact mention of the mass action law, without any justification or even a specific reference, indicates that Henderson assumed this law to be common knowledge. The second Henderson paper [3], on "The theory of neutrality regulation in the animal organism," is equally clear, and states in its second paragraph:

> In the ionization reaction of a weak acid, $\mathrm{HA} = \overset{+}{\mathrm{H}} + \bar{\mathrm{A}}$, the conditions of equilibrium are defined by the equation of the concentration law,
>
> $$k \cdot (\mathrm{HA}) = (\overset{+}{\mathrm{H}}) \cdot (\bar{\mathrm{A}}),$$
>
> in which the enclosed quantities stand for concentrations, and *k* is the ionization constant of the acid. This equation may be more conveniently written as follows:
>
> $$(\overset{+}{\mathrm{H}}) = k \times \frac{(\mathrm{HA})}{(\bar{\mathrm{A}})}$$

And two years *earlier*, in a paper on "Equilibrium in solutions of phosphates," again in its second paragraph, Henderson [4] had stated

> Before the introduction of the modern theory of solution and the concentration law, facts concerning equilibrium in solutions were necessarily stated otherwise than they are today, and conceptions regarding such systems were inevitably somewhat befogged.

Henderson earned his BA magna cum laude from Harvard in 1898, then entered the Harvard Medical School, and finally spent two years (1902–1904) with Hofmeister in Strassburg before returning to Harvard, initially as a lecturer in biological chemistry [5]. Already as a Harvard undergraduate, Henderson had submitted for a Bowdoin Prize an essay on Arrhenius' theory of electrolytic dissociation [6], published a decade earlier in the first volume of the *Zeitschrift für physikalische Chemie: Stöchiometrie und Verwandtschaftslehre*. In that paper, Arrhenius already quotes some of the relevant papers concerning the mass action law, including those by Ostwald, Guldberg, and van 't Hoff.

Arrhenius, of course, was well acquainted with the work of Guldberg and Waage. The following quote, taken from Arrhenius' highly readable Willard Gibbs address [7], indicates how well-known and accepted was the mass action law of Guldberg and Waage around 1888, fully twenty years before the quoted Henderson papers:

> I came to van 't Hoff in 1888. On the way I visited Kiel—midway between Stockholm and Amsterdam—van 't Hoff was at Amsterdam at that time—and then I spoke with Planck; he was very much interested in this subject, and he said: "I agree wholly with you, but there is a difficulty. If I consider the conductivity of copper sulfate, I may calculate how great a part of that salt is dissociated and then this part must conform to the law of equilibrium which was announced by Guldberg and Waage. The difficulty is that my calculations do not agree with that law." He asked me if it was possible to suppose that this discrepancy was due to the circumstance that the equilibrium was not reached immediately after the solution. I said that I supposed there was no hope to help it out by such a hypothesis. Then I came to Amsterdam and when I saw van 't Hoff, he said: "The dissociation theory is very good, but can you get it to accord with the law of Guldberg and Waage? I have calculated the conductivities of potassium chloride; they do not agree." Then I said to him: "Yes, that is as you say, but you ought to try the calculation with acetic acid; for with acetic acid you can change the degree of dissociation in the proportion of 1 to 100, but with the salt you cannot change it more than in a proportion of 3 to 4. Of course, you must, if you want to see if a rule is true, take the greatest variation possible." He said: "That is true, please sit down and calculate it yourself" and he gave me a logarithm table and I sat down and the result looked very well, and I showed it to him, and I said: "You see that is not so bad." He was very much interested; but he wished to have better agreement. He said to his assistant Reicher: "You must make absolutely pure acetic acid and some other acids. I will need them." Reicher did not understand why van 't Hoff needed the acids. He distilled them, but van 't Hoff was not content until after some weeks of hard work. The measurements on the new, pure preparations agreed perfectly with the theory of Guldberg and Waage. During this time he received a paper in the *Zeitschrift für physikalische Chemie*, and in it Ostwald announced that he had found that the said rule holds, and therefore it is called "Ostwald's law of dilution." You will see from this example that the idea was, so to speak, in the air. Ostwald worked on it and Planck and van 't Hoff worked on it simultaneously; Ostwald was the editor of the *Zeitschrift*, and used the opportunity to publish it first. He made, at first, no new, more accurate experiments, as van 't Hoff did, but was content with his old figures. Later, he made a great number of new determinations; he found the law of equilibrium to be true in a very much higher degree than this law had been proven before, for gases.

The people figuring in the above account were no slouches, but were, rather, among the giants of chemistry of those days. J. H. van 't Hoff was the very first Nobel laureate, in 1901, interestingly "in recognition of the extraordinary services he has rendered by the discovery of the laws of chemical dynamics and osmotic pressure in solutions" rather than for his discovery of the tetrahedral structure of carbon. Waage had died in January 1900, before any Nobel prizes were awarded, and Guldberg followed him in January 1902, five weeks after van 't Hoff got the honor. Nobel prizes are not given posthumously.

Arrhenius earned the Nobel prize in 1903 "in recognition of the extraordinary services he has rendered to the advancement of chemistry by his electrolytic theory of dissociation", and Ostwald in 1909 "in recognition of his work on catalysis and for his investigations into the fundamental principles governing chemical equilibria and rates of reactions". In their Nobel lectures, van 't Hoff, Arrhenius, and Ostwald all specifically mentioned Guldberg and his work [8]. Planck went on to develop the theory of the liquid junction, and (with Nernst, another Nobel laureate) the theory of ionic motion in solution, before focusing on the black body radiation and developing the concept of a quantized energy, for which he earned the 1918 Nobel prize in physics.

Ostwald's dilution law [9] and the work of van 't Hoff and Reichert [10] were published in 1888. In the following year, Ostwald determined and listed the K_a values of no less than 243 acids [11–13].

The work of Guldberg and Waage [14, 15] was not only common knowledge among the leading German chemists in 1888, and the sole topic of volume 104 of Ostwald's *Klassiker der exakten Wissenschaften* [16], but was also mentioned, for example, in the 1898 *textbook* on inorganic chemistry [17] by the influential Ira Rensen (then professor at Johns Hopkins and, subsequently, its president), and in the 1907 *history book* by Pattison Muir [18].

A century later it might be difficult to ascertain what Henderson knew about the mass action law, and when he knew it, but in this particular case we can be quite certain, because this law was mentioned specifically in his very first paper [19], which, incidentally, was published in German, in the very same *Zeitschrift für physikalische Chemie*, founded and edited by van 't Hoff and Ostwald, that had featured the famous Arrhenius paper as well as some of Ostwald's most extensive experimental confirmations of the Guldberg-Waage law as applied to weak acids [11–13]. In their preliminary communication [19], which was never followed up by a final account, Gordon, Henderson, and Harrington reported on measurements on solutions of $CdSO_4$ in aqueous NaCl, and their failure to find agreement between dissociation constants calculated from conductivity measurements and those computed from the mass action law:

> Diese Überlegungen zeigen wieder einmal deutlich die ganze Nichtüberstimmung in den Dissociationswerten der

> meisten anorganischen Saltze und Säuren, wenn sie aus der elektrischen Leitfähigkeit einerseits und den Folgerungen des Massenwirkungsgesetzes anderseits berechnet werden.

(These considerations again show clearly the total disagreement between the dissociation constants of most inorganic salts and acids as calculated either from the electrical conductivity or from the consequences of the mass action law. *Translation by RdL.*)

These conclusions applied to an analysis in quite concentrated (1.64 and 1.85 M) chloride solutions, in which complex *ions* such as $CdCl_3^-$ and $CdCl_4^{-2}$ were not considered, although the *molecules* Na_2CdCl_4 were. It is therefore hardly surprising that they did not find agreement between their observations and the mass action law, but for our present purpose it is important only that they clearly knew and used that law in 1898.

Henderson's 1908 papers were obviously not about the mass action law itself (or, as Henderson called them, the "mass law" or the "concentration law"), but about a specific application thereof. What, then, was Henderson's contribution in his quoted papers [2, 3] on buffer action? It is clearly stated in its next paragraph of reference 2,

> We may therefore write the equation,—
>
> $$(\overset{+}{\mathrm{H}}) = K \cdot \frac{\mathrm{HA}}{\mathrm{MA}}$$
>
> where *K* is the ionization constant of the acid divided by the degree of ionization of the salt, and *HA* and *MA* represent the amounts of acid and salt present in the solution.

In his second 1908 paper, Henderson [3] was even more explicit:

> If an aqueous solution of such an acid and its sodium salt be prepared, it is evident that here the concentration of the undissociated molecules of the acid (*HA*) will be almost precisely equal to the total amount of acid present, for such an acid under these conditions will be hardly at all ionized. On the other hand, the concentration of the ion $(\overset{-}{\mathrm{A}})$, will be equal to the concentration of the salt multiplied by its degree of ionization. This last quantity varies with the nature of the ionization of the acid and with the concentration; it is also affected by the presence of other salts in solution. For present purposes, however, it may be estimated as 0.8 without seriously impairing the conclusions. This estimate can hardly be more than 10 percent to 15 percent in error in any of the solutions here discussed.
>
> We may then write, as an approximation sufficiently accurate for our present purposes, the equation
>
> $$(\overset{+}{\mathrm{H}}) = \frac{k}{0.8} \times \frac{\mathrm{HA}}{\mathrm{NaA}};$$
>
> that is to say, in the solution of a weak acid and its sodium salt, the concentration of hydrogen ions is equal to the ionization constant of the acid divided by 0.8, approximately the degree of ionization of the salt, and multiplied by the ratio between the total amounts of acid and salt present in the solution.

Obviously, Henderson's contribution was to replace the *concentrations* of *HA* and A^- by the *total amounts* of acid and salt as if these were merely diluted upon dissolution, that is, by what we now call their total analytical concentrations, C_a and C_b respectively, where a and b identify the two components of a conjugate acid–base pair. Today we would write this as

$$[H^+] \approx \frac{K_a C_a}{C_b}$$

As Henderson explicitly acknowledged in the above quote from reference 3, this is an *approximation*. Nowadays, we might show its nature by comparison with the exact result, known as the Charlot equation [20], and directly derivable from the mass action law,

$$[H^+] = \frac{K_a (C_a - \Delta)}{(C_b + \Delta)}$$

where $\Delta = [H^+] - [OH^-]$. Such a comparison [21, 22] shows that the Henderson approximation is only useful when $C_a + C_b \gg |\Delta|$ and $C_a/C_b \approx 1$. The apparent contribution of Hasselbalch [23], the logarithmic form of Henderson's approximation, is of course entirely peripheral to this matter, and has merely raised the level of triviality in assigning names to one of the most important and well-established laws of chemical equilibrium.

It is evident from Henderson's own writings that he was not only quite familiar with the mass action law, but also assumed such familiarity on the part of his readers. To suggest that the mass action law was not generally known among chemists at the time of Henderson's 1908 paper, and that "it fell to a medical doctor to recognize the simple relationship between a weak acid, its salt, and the hydrogen ion concentration" [1] is clearly at variance with the historical record.

Guggenheim [24] has pointed out that the mass action law did not emerge fully formed, as Pallas Athena did from the head of Zeus, but included contributions by Horstmann and van 't Hoff. In addition, the experimental studies of Thomsen [25] and especially of Ostwald [9, 11–13, 26–29], and the theoretical support from van 't Hoff [30], were crucial to its rapid acceptance. Outside the US it is usually known by the names of the Norwegian brothers-in-law who first stated explicitly that concentrations rather than amounts were the appropriate quantities to use in the mathematical expressions describing chemical reaction rates and equilibria, and that these concentrations occurred in product form [14]. The mass action law found its final form by 1879 [15], at which time Henderson (1878–1942) was one year old, and Hasselbalch (1874–1962) all of five.

The mass action law was developed for reactions and equilibria of *molecules*. It was only after Arrhenius established the presence of ions as stable species in electrolyte solution that it could be applied to acid–base equilibria. That matter was essentially settled two years later, with the publication by Ostwald of 243 K_a values for a great variety of acids [11–13].

As the quote from Arrhenius illustrates, the Guldberg-Waage law was not the final word and could account for the major effects (those ranging over a factor of 100) but not for some minor details (involving a factor of 4/3). We now understand why: the concept of an ionic activity coefficient had yet to be developed. Not only was the Debye-Hückel theory [31] still many years off, but even its two foundation stones, the work of Milner [32, 33] on ionic interactions and that of Gouy [34] on the electrical double layer, had not yet been published by 1908.

As is obvious from the above, the mass action law of Guldberg and Waage was well-established by 1888, well-known to Henderson in 1898, and indeed needed no more reference in 1908. Henderson [2, 3] was very clear on the approximation he proposed, and on its specific purpose: understanding the preservation of neutrality by buffer action of a weak acid with a K_a of the order of $\sqrt{K_w}$, where K_w is the ion product of water. Henderson never claimed to have discovered the mass action law, nor should the Henderson–Hasselbalch equation be mistaken for "the mass action expression cast in logarithmic format" [1]. There is no need to confuse the general mass action *law* of Guldberg and Waage, written in terms of the concentrations of the species participating in the equilibrium, with the Henderson *approximation* for a single monoprotic buffer mixture written in terms of total analytical concentrations. The two are similar only in form, $[H^+] = K_a [HA]/[A^-]$ and $[H^+] \approx K_a\, C_{acid}/C_{salt}$, respectively, but quite different in both meaning and range of applicability.

It is not clear why some textbook writers, apparently starting with biochemists but subsequently including some chemists and even an occasional analytical chemist, have incorrectly attached the names of Henderson and Hasselbalch to the mass action law. Henderson and Hasselbalch only considered an approximate result for the specific solution of a single weak monoprotic acid and its conjugate base, whereas Guldberg and Waage, whose names are now largely unknown in this country, four decades earlier had formulated a much more general law [35]. But the confusion persists, and only adds to the difficulty students have with the mass action law and its quantitative applications.

References and Notes

1. Po, H. N.; Senozan N. M. *J. Chem. Educ.* **2001,** *78,* 1499–1503.
2. Henderson, L. J. *Am. J. Physiol.* **1908,** *21,* 173–179.
3. Henderson, L. J. *Am. J. Physiol.* **1908,** *21,* 427–448.
4. Henderson, L. J. *Am. J. Physiol.* **1906,** *15,* 257–271.
5. Cannon, W. B. Bibliographical Memoirs of the National Academy of Sciences of the USA **1943,** 23, 31–58.
6. Arrhenius, S., *Z. Physik. Chem.* **1887,** *1,* 631-–648.
7. Arrhenius, S. *J. Am. Chem. Soc.* **1912,** *34,* 353–364.
8. Nobel lectures Chemistry 1901–1921; Elsevier: Amsterdam, 1966.
9. Ostwald, W. *Z. Physik. Chem.* **1888,** *2,* 270–283.
10. van 't Hoff, J. H.; Reicher, L. Th. *Z. Physik. Chem.* **1888,** *2,* 777–781.
11. Ostwald, W. *Z. Physik. Chem.* **1889,** *3,* 170–197.
12. Ostwald, W. *Z. Physik. Chem.* **1889,** *3,* 241–288.
13. Ostwald, W. *Z. Physik. Chem.* **1889,** *3,* 369–421.
14. Guldberg, C. M.; Waage, P. *Forhandl. Videnskabs-Selskabet Christiania* **1865,** 35-45. A facsimile as well as an English translation can be found in *The law of mass action;* Haraldsen, H., Ed.; Universitetsforlaget: Oslo 1964.
15. Guldberg, C. M.; Waage, P. *J. Prakt. Chem.* **1879**, (*2*) *19,* 69–114.
16. Klassiker der exakten Wissenschaften 104; Ostwald, W., Ed.; Leipzig, 1899.
17. Remsen, I. *Inorganic Chemistry;* Henry Holt & Co.: New York, 1898.
18. Pattison Muir, M. M. *A History of Chemical Theories and Laws;* John Wiley & Sons: New York, 1907.
19. Gordon, C. M. C.; Henderson, L. J.; Harrington, W. L. *Z. Physik. Chem.* **1898,** *29*, 425-428.
20. Charlot, G. *Anal. Chim. Acta* **1947,** *1*, 59-68.
21. de Levie, R. *Principles of Quantitative Chemical Analysis;* McGraw-Hill: New York, 1997; p 135.
22. de Levie, R. *Aqueous acid–base equilibria and titrations;* Oxford Univ. Press: Oxford, 1999; pp 50–51.
23. Hasselbalch, K. A. *Biochem. Z.* **1916,** *78,* 112–144.
24. Guggenheim, E. A. *J. Chem. Educ.* **1956,** *33,* 544–545.
25. Thomsen, J. *Ann. Phys. Chem.* **1869,** *214,* 65–102.
26. Ostwald, W. *J. Prakt. Chem.* **1877,** *124,* 385.
27. Ostwald, W. *Ann. Phys. Chem.* **1878,** *8,* 154–168.
28. Ostwald, W., *J. Prakt. Chem.* **1879,** (2) *19, 468* (2), 468–485.
29. Ostwald, W. Die wissenschaftlichen Grundlagen der analytischen Chemie; Steinkopf: Dresden, 1894.
30. van 't Hoff, J. H. *Études de dynamique chimique;* 1884.
31. Debye, P.; Hückel, E. *Phys. Z.* **1923,** *24,* 185–206.
32. Milner, S. R. *Phil. Mag.* **1912,** *23,* 551–574.
33. Milner, S. R. *Phil. Mag.* **1913,** *25,* 742–751.
34. Gouy, G. *J. Phys.* **1910,** *9* (4), 457–468.
35. de Levie, R. *J. Chem. Educ.* **2000,** *77,* 610–612.